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Exercises: Charlotte Amann-Bartnick

Albert-Ludwigs-Universität Freiburg
Winter term 2025/2026

Model Theory

Sheet 7

Deadline: 04.12.25, 2:30 pm.

Exercise 1 (6 points).

In the language $\mathcal{L} = \{<\} \cup \{c_n \mid n \in \mathbb{N}\}$ consisting of infinitely many constant symbols c_n together with a binary relation symbol $<$, consider the $\mathcal{L}$-theory T of dense linear orders without endpoints such that in every model $\mathcal{A}$ the sequence $(c_n^{\mathcal{A}})_{n\in\mathbb{N}}$ is strictly increasing.

a) Show that T is complete with quantifier elimination.

b) Show that T has only three countable models up to isomorphism. These models can be choosen with universe $\mathbb{Q}$.

Hint: $\lim_{n\to\infty} c_n^{\mathcal{A}}$

Exercise 2 (8 points).

Let $\mathcal{L}$ be the language consisting of infinitely many unary predicates $(P_i)_{i\in\mathbb{N}}$. Consider the class $\mathcal{K}$ of all $\mathcal{L}$-structures $\mathcal{A}$ such that for all finite disjoints subsets I and J of indices in $\mathbb{N}$ there exists infinitely many a's in A which lie in every $P_i^{\mathcal{A}}$ for all i in I but do not lie in any $P_j^{\mathcal{A}}$ with j in J.

a) Give an axiomatization T of $\mathcal{K}$ and show that T is consistent.

Hint: Random graph.

b) Show that T is complete with quantifier elimination.

c) Describe (informally) all 1-types in $S_1(T)$.

d) Is T totally transcendental? Does T have a prime model?

Exercise 3 (6 points).

Let $\mathcal{A}$ be a countable model of an $\aleph_0$-categorical theory T in a given countable language.

a) Show that every partial elementary map $f : A_1 \to A_2$ between finite subsets of $\mathcal{A}$ extends to an automorphism of $\mathcal{A}$.

b) In particular, whenever two tuples $\bar{a}$ and $\bar{b}$ in $\mathcal{A}$ realize the same type over a finite subset C, show that there is an automorphism of $\mathcal{A}$ fixing C pointwise and mapping $\bar{a}$ to $\bar{b}$.

c) Let $1 \leq n$ be in $\mathbb{N}$ and consider a subset X of A^n which is invariant under all automorphisms of $\mathcal{A}$ (i.e. every automorphism α only permutes the elements of X, but fixes X setwise). Show that X is definable without parameters.

Hint: Ryll-Nardzewski

The exercise sheets can be handed in in pairs. Submit them in the mailbox 3.19 in the basement of the Mathematical Institute.